

Pre-school Students' Informal Acquisitions Regarding the Concepts of Point and Straight Line

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Abstract

This study aimed to investigate the informal cognitive structures regarding "point" and "straight line" -two basic and undefined terms of geometry- in children registered in preschool – the previous step before in-class formal education process. The study was conducted with the participation of 50 children enrolled in nursery, kindergarten and preschools of a total of five educational institutions -three public and two private- in a city which is in the middle of the Turkey. The qualitative research model was utilized in the study since observing, analyzing and assessing children's intuitive thinking and informal knowledge construction process would be difficult and good results would not be obtained via quantitative research methods. Data were collected through clinical interview technique. Results show that children, in general, possess major and to a large extent correct acquisitions that would be the basis of subsequent formal concept development process in children.

Keywords: Qualitative research, clinical interview, informal acquisition, point, straight line.

Introduction

In Mathematics teaching, it is known that children's intuitive learning especially in preschool and first years of primary school provide an important foundation for future learning. Common opinion of many researchers (Ausubel, Gagne, Piaget and others) who generated the theoretical background related to this issue is: "when suitable learning environments are created, it is easier for the cognitive acquisitions (cognitive structures) obtained through children's intuitions to construct and transfer knowledge. Piaget argues that intuitive thinking starts at the beginning of year four (Ülgen, 1999). It is well known that modern mathematics teaching approaches are usually shaped based on this fundamental view.

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Piaget stated that children construct knowledge themselves and this process called "adaptation" includes various sub-processes (assimilation, accommodation and equilibration) (Altun, 2010). In his studies related to concept formation, Piaget emphasized that analysis of the existing informal acquisitions should precede the formal formation process of a concept and stated that children's formal acquisitions are built upon these informal acquisitions (Altun, 2010; Günçe, 1973). Piaget also remarked that children construct knowledge on their own based on their own anatomical structures and through interactions with the environment and that especially language development and concept development go hand in hand (Ülgen, 2001).

Based on constructivist approach, current study aimed to observe the existing informal cognitive structures in children regarding the concepts of point and straight line -the two basic and undefined concepts of geometry- during the preschool period which is provided immediately before in-class formal education process and to discuss and interpret the contributions of this structure to formal education and training process.

According to Piaget and various clinical psychologists, it is rather hard and sometimes impossible to reconstruct misconceptions in children's cognitions (Ülgen, 2001). Children may not be able to ensure equilibrium between misconceptions and the correct constructs presented in the classroom and may face dual concept formation (one created by the children and one presented to the children), misconceptions and incomprehensibility in the future (Ülgen, 2001). Therefore, the current study hopes to observe possible misconceptions in children as well. It is necessary to point out here that this study does not intend to comprehend how cognitive constructs related to point and straight line concepts are generated in children. The study focuses on and is limited to observing and assessing the existing constructs.

Any scientific research –regardless of its field– is conducted with the help of a specific method or more than one method that complements one another. Various classifications are possible for different types of scientific research. One of these classifications distinguishes types of research as "descriptive", "relational" and "experimental" (Karakaya, 2011). Two major research methods in the field of mathematics teaching are "quantitative research" and "qualitative research" methods. Quantitative research is generally based on numerical data about mathematical knowledge and skills and is used to determine level/degree. On the other hand, the qualitative research method is used to observe the existing states in individuals and obtain and interpret general impressions that are not based on numerical data (Cemaloglu, 2011).

Qualitative Research Model

Current study is a descriptive qualitative research and information obtained from the related literature regarding the appearance, development, principles and techniques (alternative methods) of qualitative research method and suggested action research types are provided below in a partly chronological manner. The research is carried out in accordance with this model.

Paul Ernest defines qualitative research method as an important method whose developmental process is ongoing, uses the term "model" in the place of "method" and states that qualitative research method was first used in social research. He also points that qualitative research was first shaped in mathematics education research by Erlwanger (Ernest, 1998).

Ernest addresses the developmental process of qualitative research model as composed of three periods: "rationalist period", "modern period" and "post-modern



period". He also argues that rationalist period started with Descartes and the modern period commenced with Piaget (Ernest, 1998).

Piaget started his qualitative research in 1920's (Ginsburg, 1981). Piaget employed a clinical interview technique in his research to comprehend small children's cognitive constructs of and how knowledge is constructed in the minds of children. For long years, Piaget used standard tests that utilized this technique but after the 1950's, he tried to integrate some flexibility in the technique (Ernest, 1998; Ginsburg, 1981).

Starting with 1970's, this model and post-modern research that highly contributes to the development of epistemology are more prominent. Erlwanger, Rorty, Gardes, Gardner, Zoslovsky, Ginsburg, Ernst von Glasersfeld, Erickson, Croswell, Goldin, Silverman and Kilpatrick can be cited among the prominent researchers in the post-modern era. Some of these studies focused on "psychology of learning" whereas others focused on "problem-solving" (Baki, Karataş & Güven, 2002; Ekiz, 2004; Ernest, 1998; Ginsburg, 1981).

Clinical Interview Technique

The clinical interview is regarded as the most suitable technique to observe and interpret existing acquisitions of small children and suggest future measures. However, it is stated that this technique is rather difficult and risky; it requires diligence in terms of reliability and validity and in order to increase validity and reliability, it is necessary to employ the technique by freeing it from obligatory standards and to administer it in a flexible manner in the form of semi-structured activities (Çepni, 2007; Ekiz, 2004; Ginsburg, 1981). Therefore, the sequence of experiences for the activities should be the same for each interviewer but questions and stimulants should be systematic and somewhat impromptu according to the atmosphere, children's desire for synergy and the developed empathy and dialogue. Success mostly comes from researcher's ability to empathize and his/her research experiences (Ekiz, 2004; Ernest, 1998).Activity instructions and observation forms should be prepared beforehand for clinical interviews and sequential experiences should be realized by "working together" with children and should be immediately recorded. According to the above information, this type of activity can be cited as "structured fieldwork" (Cemaloğlu, 2004).

The items below should be taken into consideration during the interviews in the light of the relevant literature:

- Doing interviews while seated at a table may not provide best results. Therefore, children's natural environments (floor, play corner, etc.) should be preferred. Children may present some cognitive skills in their natural environments while they may not be able to display them in artificial environments (such as interview rooms) (Ginsburg, 1981).
- Interviews should be carried out in accordance with the anatomical structure of the children and their current affective states (excitement, use of the left hand, style of using materials etc.) (Ernest, 1998; Goldin, 1998).
- It is possible for children to provide unexpected, surprising and interesting answers during interviews. In such cases, it is necessary to focus on the answer during the interview of in the upcoming interviews and impromptu questions should be carried in an order of increasing accuracy to comprehend the actual cognitive constructs (Bacanak, 2008).



- Children should be given ample time to give their answers and to present their answers by doing whereas no interventions should be provided other than small tips (if necessary).
- Children should feel free to ask questions when they do not understand the questions and when they need clarification or explanations.
- When the children are unwilling to work together, when they feel disinterested in any part of the interview or when they lose motivation, it is not wise to commence or continue the interview.
- When a question is unanswered, the interviewer can move on to next question provided that willingness and motivation still exist (Ginsburg, 1981).

Method

Based on the rationale presented in Introduction, it was decided to undertake the study in the form of one-on-one interviews with the children in an appropriate environment and to use written observation forms only with the concern that voice recording or video recording may distract 4-6 year-olds although voice or video recording is generally suggested (Bacanak, 2008; Clarke, 1998; Ekiz, 2004).

Creating the Hypothesis

Gestalt Theory which was started at the beginning of 1990's by Wertheimer, Kafka and Köhler; and was evolved after the years of 1990's especially in America and has still been evolving is a teaching and learning theory that was formed against Behaviorist's warning response theory. This theory is also against Constructivist Theory's in-depth analysis that reduces to elements of mind (Senemoglu, 2002; Yıldırım, 2008; Schunk, 2009). Gestalts advocate that introspection method of structuralists is a suitable method for examine learning case however, it is used wrong. They assert that it is better to handle mental experiences organized as a whole instead of analyzing segmentally (Senemoglu, 2002).

As discussed in the introductory part, the observation of a configured shape as intuitively is targeted, not the way of how the concept of points and lines is shaped in children. It is understood that it is configured the point as a small circular track and the line as a flat and solid line. In addition, this situation can be observed and analyzed at a satisfactory level with using proper materials.

It was supposed that "children between the ages of 4–6 may have informally perceived the point as a very small circular trace and the straight line as a straight and unbroken line". Based on this hypothesis, materials to work on point and straight line concepts and "instructions" and observation forms" for interviews were created.

Materials

For working on the concept of point: blank papers, papers presenting single colored very small points (zero-dimensional), as a distractor somewhat larger points (small filled circles), small scale circles, filled or empty triangles, alike squares and very short lines.

For working on the concept of straight line: blank and lined (squared) white papers, papers presenting black colored lines, curves, broken lines, disconnected lines etc., strings, rubbers and short bent plastic rods (3–4 cm).



Instructions

Sequential experiences designed for the concept of point:

- The child is provided with a blank paper and pencil and asked to form points on the paper. The child is asked to compare actual formations by drawing attention to possible wrong formations (Which one is better? Which one looks more like a point? Is it like the one you make with your finger? etc.)
- The paper that includes points and distractors are placed on the table and the child is asked to point to the points. Possible wrong selections and correct selections are compared and questions are directed about the differences between them (Does it look like the one you drew with the pencil? What is the difference? Can you try one more time? etc.)
- The researcher draws a simple shape on a blank paper (triangle, circle, square etc.) and the child is asked to create a similar one by using points.
- The child is congratulated at the end of the activity. The child is expected to answer questions such as "Yes, so what is a point?", "What does it look like?" and to point to the points.
- The researcher records his/her observations and the dialogs for each sequential experience in the observation form with care and diligence.

Sequential experiences designed for the concept of straight line:

- A similar discussion platform is created by mentioning the term "straight line".
- The child is provided with a blank paper and pencil and asked to form/draw several straight lines on the paper. The child is asked to compare actual formations by drawing attention to possible wrong formations (Do you think that part of the line is fine? Can you see the difference between them? How would you draw the best one if you did it again? Shall we do it again? etc.). Also, the reactions are recorded.
- The child is provided with a lined (squared) blank paper and asked to draw several straight lines. Whether the child takes the lines on the paper as a reference is noted and recorded.
- A white colored paper with straight lines, curves, broken and disconnected lines is presented to the child and he7she is asked to select the correct ones among the lines. A dialog similar to the one experienced for the concept of points is generated for possible wrong selections. If all selections of the child are correct, he/she is asked to make comparison with a sample that is wrong (For instance, why did not you choose this one? What would you say if I selected this one? etc.). Based on the answers, question-answer session continues.
- The child is provided with a piece of string and asked to form a line using the string. He/she is asked to do the same with the rubber. Whether the child uses both hands and whether the materials are stretched is observed for both materials. If the child is not stretching the materials sufficiently, he/she is asked to do so and asked questions about the difference between both cases. The researcher may intervene when the string is stretched and asks the difference between the two conditions (What happened now? What should we have done? What should I do? What happens if I do this? etc.). Then the researcher presents 6-7 rods with some curved parts and asks the student to use three rods to make a straight line by placing them side by



side. Questions are asked about possible wrong selections (Is this rod OK? Is it fine now? What should we do? etc.)

- Finally, last ideas are collected with questions such as "Yes, so what is a straight line?" and "How do we make a straight line?" If necessary, actions are used to present the ideas.
- Observations and impressions during the sequential experiences are recorded in the observation form in the same way it is handled for the point concept.

Testing the Hypothesis

Based on a consensus with their teachers, 10 children (one female and one male student from each school) between 50-70 month chronological age were selected from the nursery, kindergarten and pre-school classes of the five schools. The schools were contacted beforehand to obtain necessary permits for the study.

Two researchers visited the identified schools. The study was conducted with the selected students in a separate location (in a separate corner of the class) away from the rest of the classroom and necessary notes were taken. Based on the impressions obtained during the hypothesis testing phase, required changes in the instructions for the general implementation, the manner of getting together with the children at the beginning of the study and the necessary actions to determine effective readiness (suppress excitement, increase curiosity, generate willingness etc.) were identified.

Since satisfying levels of empathy were established with the children during test hypothesis phase, researchers felt that the test was successful and experienced selfesteem for the actual implementation. It was also believed that experienced gained in a similar qualitative research (Develi & Orbay, 2002) would support the implementation.

Establishing the actual working group

20 children who completed year 4 and 20 children who completed year 5 were selected based on teacher views from the identified schools by taking their chronological age into consideration and ensuring balanced gender distribution. In each school, all the children with prior nursery school experience were included in the working group. Instructions for the activities were reorganized in line with the observations obtained during the hypothesis testing phase. Materials were improved and finalized for the implementation.

Results

The implementation was carried out in 10 working days in the identified schools. Two researchers worked in separate environments by dividing the number of children among themselves. Insufficient motivation, problems in the flow of sequential experiences due to various reasons, negative reactions (sudden silences, shrugging etc.) and unwillingness were observed just a few times (in 4 children). Interviews were stopped with these children and other children were included in their places. Almost all the children started the interviews with high motivation at first maybe due to the promised reward. All children behaviors during interviews, their surprising comments, interesting dialogs that took place and their drawings were collected diligently for assessment. 36 of the 40 children that participated in the implementation provided fun and challenging, surprising and interesting dialogs and displayed amazing actions. In order to clarify the data analysis, some of these dialogs were given as examples below and the first example was detailed.



Example 1

Researcher: K.O.

Child friend: Z. B. G.

K.O.: Welcome, my friend (they shake hands). My name is K.O. Can I learn your name?

Z.B.G.: Z.B.G.

K.O.: Z.B., now we are going to undertake a very entertaining activity with you. I believe we will be successful. You know there is a reward at the end!

Z.B.G.: OK!

K.O.: Z.B., can you tell me what a point is?

Z.B.G.: Circle

K.O.: OK, can you also tell me how a point looks like? For instance, if I ask you to show me with your finger!

Z.B.G.: The child taps the low table with the tip of is/her index finger: Tap! Tap!

K.O.: Well done! Congratulations! Now can you make a few points on this paper with this pencil?

Z.B.G.: The child is carefully making points by using the tip of the pencil (Figure 1).



Figure 1. Marking points

K.O.: Nice job! Congratulations! Now I will give you a paper. There are some marks on it. Can you show me which of these marks are points? It is sufficient to point with your finger!

Z.B.G.: OK! (The child points to all objects shaped like the following "., ,, o".)

K.O.: "If I told you to select only one of them", which one would you choose as the point?

Z.B.G.: The child selects the ".".

K.O.: Why did not you select "o"?

Z.B.G.: Because it is blank inside.

K.O.: Then why did not you select "."?

Z.B.G.: It is very big!

K.O.: We are getting closer to the end of the point work, Z.B.! Now I will draw a figure for you. Can you do the same for me with points, I mean by using points?

Z.B.G.: I can!

K.O.: Here is a figure for you. Do it and we will see!



Z.B.G.: Hmm, this is a triangle! It is easy! (The child finishes Figure 2 in a short time)



Figure 2. Drawing triangles by using points

K.O.: Great work! So, what is the point? How is it then?

Z.B.G.: Just like that! (The child shows the shapes on the paper other than "o" and touches them with the tip of her finger)

K.O.: Applauds from me! Congratulations! You are very successful! Now let's rest for a while. We will undertake another small activity shortly. It won't take a lot of your time! Your reward is waiting for you! (They rest)

K.O.: Now let's move on to the straight line! Are you ready?

Z.B.G.: Yes.

K.O.: Z.B., what is a straight line?

Z.B.G.: The child does not answer. He/she shrugs.

K.O.: OK. Z.B., what is a line then? How is it made?

Z.B.G.: Hmm, that! The child draws a line on the floor that looks close to a straight line.

K.O.: Dear Z.B., how is a very straight line then?

Z.B.G.: The child draws a line with her finger more carefully.

K.O.: Dear Z.B., the last line you drew, the one that is "very straight", is called a "straight line".

Z.B.G.: You mean a road!

K.O.: Very true! Bravo! Now, draw a few straight line son this paper with the pencil!

Z.B.G.: Ooo it is very easy! (The child draws the vertical line provided below, Figure 3).

K.O.: Now let's make another one like this (the researcher roughly points to the horizontal line).

Z.B.G.: The child draws the line on the paper (provided above) (he/she does not use the lines on the lined paper as a reference, Figure 4).



K.O.: Bravo! Bravo! Now I will show you another paper that looks like the previous one. I will ask you to select the "straight lines" from the figures.

Please pay attention dear Z.B.! (The child mostly pointed to the straight lines however he/she also marked ______ those)



K.O.: Nice! Can you also do the same with the rubber?

Z.B.G.: Yes, it is easy! (The child stretches the rubber.)

K.O.: We are almost done! One last experiment! (The researcher takes out 6-7 plastic rods some of which are curved). Come on! I want you to select 3 of these and connect them like this (displays with both fingers) to make a straight line. You have only three options! Pay attention, good luck.

Z.B.G.: ______ (the child makes the selections)

K.O.: Don't you think there is something wrong here? What do you say?



Z.B.G.: Ha, yeees, the one in the middle does not fit!

K.O.: OK. Now, select something else instead of it.

Z.B.G.: The child makes the correct selections and places them where they belong.

K.O.: Good job! So, what is a straight line then?

Z.B.G.: It is a very straight line, like this (Shows with hands)!

K.O.: You are a very successful child. Thanks for working with me! Would you like a chocolate Miss Z.B.? (They share laughter, researcher pats her head and they say goodbye)

Example 2

Researcher: H.D.

Child friend: D.M.

H.D.: Can you make a shape that is similar to the one I will draw by using points?

D.M.: Yes! Like that. (Figure5)



Figure 5. Shape

H.D.: Why did not you select this? (Pointing to /) D.M.: Because it is wavy. Example 3 K.O.: Why did you select this? (Pointing to –) -0-Z.İ.: Because they are the most beautiful ones! K.O.: Why? Z.İ.: Looks like a necklace! Example 4 H.D.: Why did not you select this? (Pointing to -) Y.Ö.: Zigzags won't do! Example 5 H.D.: What is a straight line?



K.D.: It means it is not wrong! (Confuses the terms since Turkish word for straight line also means true/correct)

H.D.: I did not mean that. How do we draw a straight line?

K.D.: Ooo, you mean this? Just like that (draws with finger).

Example 6

H.D.: The researcher holds the pencil to the stretched string and forms a bulge and asks, "So, it is 0k now?"

Ç.Ö.: Nooo!

H.D.: Why not?

Ç.Ö.: Because it cannot have a peak!

Data Analysis

As the interviews of the study were conducted as open-ended questions, analyze of the data was executed responsively to the extent allowed by the literature (Cemaloğlu; 2011; Çepni, 2007).

As was expected, it was observed that it required expertise to continue the implementation, from appropriate dialogues, suitably generate the question chain, comprehend what the children mean and make sense of children's gestures and facial expressions. It was observed that children left some questions unanswered although in our opinion they looked easy to understand and answer or they provided insufficient or incorrect answers to them whereas they were able to answer questions that required higher level competence with unexpected level of correctness and meaning. For instance, it was observed that the child who drew the straight line as the curve was able to select the correct options from among the distracters and another child who selected the curve instead of the straight line was able to identify the fact that the string would be taut while making a line and implement the action as well.

Although the first step in concept development process is "informal recognition", the sequential experiences activities we designed purposefully aimed to collect children's views about point and the straight line, which can be regarded as the informal definition step. This was designed for two purposes. Firstly, it was aimed to observe the consistency between children's ideas and impressions about the concepts of point and straight line and the mechanical formation and use of those concepts during the activities in implementation and secondly, it was aimed to compare and interpret answers to "so what is a point?" and "so, what is a straight line?" provided by the children at the end of activity and their behaviors during the process. As a result of the activities designed with those purposes, it was observed that the majority of children- other than a few exceptions- was able to define the concepts correctly by pointing to the concepts instead of talking about them and they were also able to relate correct views when they were asked although their ideas did not fully cover the topic.

Children, in general, perceive the point as a very small circle object. However, sometimes they can select or perceive formations that are larger or full in the inside as points as well. This may be related to their idea of making the point more observable rather than lack of perception or misconceptions. It was observed that children did not identify geometric figures that are full in the inside, that are very small or empty in the inside or that have corners as points. Also when children with previous nursery school experiences and children who have learned the figures of triangles, rectangles, squares and circles in preschool classes until the implementation date were asked why they did



not select these figures as points, their answers included statements such as "because it is a triangle...etc" which showed that perception of the concept of point as an object with no dimension (with zero dimensions). Children were able to display expected behaviors when they were asked to form simple shapes using points with appropriate tips (form the shape by using points). This competence shaped the opinion that the children were ready to use points as basic geometrical instruments. Although there were no significant differences between children who completed year 4 and 5 in terms of recognition and use of points, it was observed that children who completed year 5 needed fewer tips and researcher support. It was seen that the majority of children were not able to answer the question "in your opinion, what is a straight line?". Probably due to the fact that this concept was not introduced as a geometrical thought during class activities at the time of the implementation. But they were able to provide expected answers to the question "what is a line?" even though they mostly pointed to lines while replying. It was identified in the interviews held with the teachers of the children that the children that participated in the implementation were able to recognize and use concepts such as "line" and "rod" during class. This information led us believe that the children actually perceive the straight line as continuous, unbroken and unbent straight line and that their hesitations at the beginning of the activity resulted from lack of familiarity with the words used to describe the concept, not with the concept itself. The majority of children were observed to be able to select the correct option from among the distractors. However, a small minority of children selected the shapes that looked nice to them. When those children were asked to compare their selections with the actual answer, they did not select the shapes that they pointed at first. When they were asked why they changed their minds, they started with their own words that they later realized the discontinuous nature of the shape ("it is broken", "there can't be hopes in it" etc.). these statements led the researchers believe that misperceptions were somewhat psychological and sometimes they resulted from carelessness. The children were successful when they were given enough tips (such as use both hands, connect by bringing both ends together etc.) during formation of the straight line with materials (string, rubber, rods). They generally drew the straight line as vertical and this may be related to the implementation of drawing number 1 as a vertical line during preschool classes while learning number 1. Children used the line of lined paper during this action. However, when horizontal lines were pointed and they were asked to "draw another one like that", they were somewhat less skillful and did not use the lines of the paper as reference. This finding points the fact that the concept of lines is perceived intuitively, but intuitive competence was not developed at the point. Broken, curved or disconnected lines were not credited with the following reasons: "this has zigzags, this won't do", "it is like the sea", "it is wavy", "it spreads out from the borders", "it is disconnected, it has holes", "it is curvy", "it is like a mountain (hill)", "it goes to the side", "it is skewed". These statements show that the majority of the children had a cognitive competence to identify the correct shape from among the distractors.

Results and Recommendations

Study results generated the view that children between the ages of 50-70 months had important and mostly correct informal acquisitions about the concepts of point and straight lines. These informal acquisitions will be the basis of future formal concept development. We believe that similar studies that will be held in similar environments will result in similar findings as well.

Significant findings were not reached during the study about how the children acquired these perceptions. It is known that, regardless of the concept, it is the hardest part of studies to comprehend and interpret informal acquisitions of children (Ernest, 1998). Therefore, as mentioned in the introduction, the study did not intend to focus on this area.



We believe that the study provides a good example to preschool teachers who undertake semi-formal training activities and especially to first grade teachers about the importance of children's informal acquisitions and the need to establish formal training on this basis. Especially while starting to teach similar concepts; it is suggested for preschool and classroom teachers to carefully use the question-answer technique similar to the one used in the study and to frequently engage in dialogues with students about children's informal acquisitions and to direct their teaching based on the impressions gained from these interviews and dialogues. It is imperative to achieve high quality in preschool education which is becoming widespread in our country.

Assessment of the semi-informal acquisition process efficiently will be the precondition of minimizing possible future misconceptions, ambiguities and formation of double concepts.

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References

- Altun, M. (2010). Eğitim Fakülteleri ve Sınıf Öğretmenleri için Matematik Öğretimi, Aktüel Alfa Yayınevi, Bursa.
- Bacanak, A. (2008). Fen ve Teknoloji Dersi Performans Değerlendirme Formlarına Yönelik Oluşturulan Web Tabanlı Programın Etkinliğinin Araştırılması, Yayımlanmamış Doktora Tezi, Trabzon.
- Baki, A., Karataş, İ., Güven, B. (2002). Klinik Mülakat ile Problem Çözme Becerilerinin Değerlendirilmesi, V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, 1043–1049, Ankara.
- Cemaloğlu, N. (2011). Veri Toplama Teknikleri Nicel-Nitel, Bilimsel Araştırma Yöntemleri, 131–164, Ed. A. Tanrıöğen, Anı Yayınları, Ankara.
- Çepni S. (2007). Araştırma ve Proje Çalışmalarına Giriş, Üçyol Kültür Merkezi Yayınları, Trabzon.
- Clarke, D.J. (1998). Studying the Classroom Negotiation of Meaning: Complementary Accounts Methodology, Monograph, National Council of Teachers of Mathematics, 9, 98–111, USA.
- Develi, M.H., Orbay, K. (2002). İşlem Öncesi Dönem Çocuklarında Sayı Kavramının Gelişimi Üzerine, V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, 969–974, Ankara.
- Ekiz, D. (2004). Eğitim Dünyasının Nitel Araştırma Paradigmasıyla İncelenmesi: Doğal ya da Yapay, www.tebd.edu.tr/arsiv/2004-Cilt2/Sayı-4/415-439pdf., Ankara.
- Ernest, P. (1998). The Epistemological Basis of Qualitative Research in Mathematics Education: A postmodern Perspective, Qualitative Research Methods Mathematics education, Monograph, National Council of Teachers of Mathematics 9 22–39, USA.



- Ginsburg, H.P. (1981). The Clinical İnterview in Psychological Research on Mathematical Thinking: Aims, Rationales, Techniques, For the Learning of Mathematics, 3, 4–11.
- Goldin, G. A. (1998). Observing Mathematical Problem Solving Through Task-Based Interviews, Qualitative Research Methods Education, Monograph, National Council of Teachers of Mathematics, 9, 40–62, USA.

Günçe, G. (1973). Çocuk Zihin Gelişimi Piaget Kuramına Toplu Bakış, Baylan Matbaası, Ankara.

Karakaya, İ. (2011). Bilimsel Araştırma Yöntemleri, Bilimsel Araştırma Yöntemleri, 55–84, ed.A. Tanrıöğen, Anı Yayınları, Ankara.

Schunk, D., H. (2009). Öğrenme Teorileri Eğitimsel Bir Bakış, Nobel Yayın Dağıtım, Ankara

Senemoğlu, N. (2002). Gelişim Öğrenme ve Öğretim Kuramından Uygulamaya, Gazi Kitapevi, Ankara.

Ülgen, G. (1999). (C.M. Charles'den çeviri). Öğretmenler İçin Piaget İlkeleri, Anı Yayınları, Ankara.

Ülgen, G. (2001). Kavram Geliştirme Kuramlar ve Uygulamalar (3. Baskı), Pegem Yayınları, Ankara.

